GUTENBERG SCHOOL OF MANAGEMENT & ECONOMICS
Johannes Gutenberg-Universität Mainz

Master of Science in
Quantitative Decision Making in Economics and Management

- Information Material about the Entrance Test -

This brochure summarizes general information about the entrance test for the QDEM program. In addition to general information, all subject areas are presented with a brief description, representative tasks and recommended literature.

The aim of the entrance test is to ensure you, that you have solid foundations in the related fields, namely mathematics, statistics and microeconomics as well as basic knowledge in computer science. These foundations are required to successfully take part in the program.

The entrance test, which counts for 50% of the procedural grade, is a 90-minutetest taking place **online**. The style of the actual test is identical to the sample questions on the following pages. Its focus is on conceptual understanding. Concrete arithmetic requires simple computations.

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I. Calculus

functions with one/many variables, differentiation, optimization (with/without constraints), integration

You should have a solid foundation in calculus as provided in a typical mathematics course in an undergraduate economics or management program. Since economics uses maths as a kind of language to express systems (e.g., demand and supply) and to find optimal solutions, the focus is on applications.

Literature:

- Essential Mathematics for Economics Analysis, K. Sudsæter, P. Hammond, A. Strøm, A. Carvajal; Pearson 2021; Chapter 5, 6, 9 and 10

Example Tasks:

(A) Consider the following function.

$$\mathrm{f(x)} = 20 + 7\mathrm{x} - 0.5\mathrm{x}^2$$

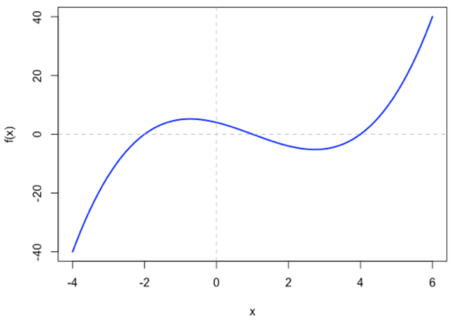
a. What is the slope of the function at $\mathrm{x} = 6$?

Now consider the range $x \in [1, 15]$

b. Provide a x-value in that range so that $f(x) = 0$.
(If there is more than one value, specify only one. If there is no such value, enter "-999" as the answer. Round to 3 decimal places if necessary.)

c. Provide a x-value maximizing $f(x)$ in that range.
(If there is more than one value, specify only one. If there is no such value, enter "-999" as the answer. Round to 3 decimal places if necessary.)

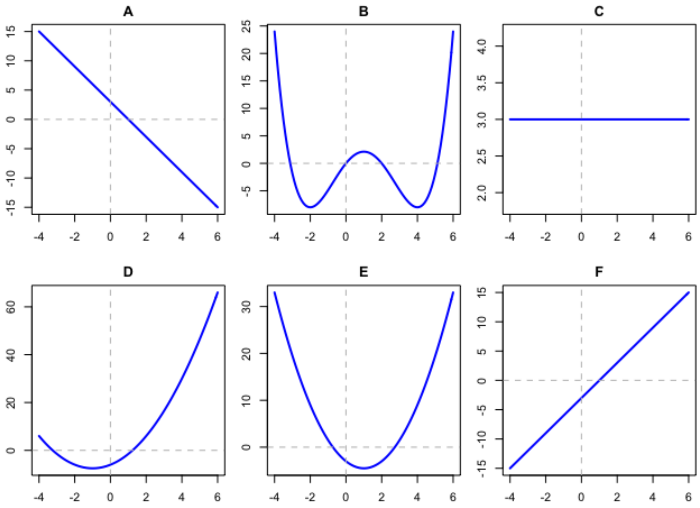
(B) Consider the following function plot.



Which plot (A-F) shows …

a. … the first derivative ($\frac{\partial f}{\partial x}$)

b. … the second derivative ($\frac{\partial^2 f}{\partial x^2}$)



II. Matrix Algebra

notation, operations, systems of linear equations

Matrix algebra is used in various setups. In statistics, for example, it is used as a compact representation of data. In operations research for the formulation and solution of linear optimization tasks. Therefore, we focus here on a solid understanding of the basic concepts as a basis for all subsequent applications.

Literature:

- Essential Mathematics for Economics Analysis, K. Sudsæter, P. Hammond, A. Strøm, A. Carvajal; Pearson 2021; Chapter 12, 13 and 19

Example Tasks:

(C) Assign the correct answer (A-G) to the task (a, b).

a.

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \times \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} =$$

b.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} =$$

(A) 32

(B) $\begin{bmatrix} 4 & 10 & 18 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & 8 & 12 \\ 5 & 10 & 15 \\ 6 & 12 & 18 \end{bmatrix}$

(E) 90

(F) $\begin{bmatrix} 4 & 0 & 60 \\ 0 & 10 & 0 \\ 0 & 0 & 18 \end{bmatrix}$

(G) no valid task

(D) Consider the transition matrix which summarizes the probabilities for switching from a status (A-C) to a status (A-C).

		to		
		A	B	C
from	A	0.1	0.3	0.6
	B	0	0.5	0.5
	C	0	0	1

a. Assume an initial distribution to the states [A: 10, B: 20, C: 0]. What is the distribution after one round?

b. What is the limiting distribution after many (∞) rounds?

III. Descriptive Statistics

features, statistical measures for uni-/bivariate distributions, visualization

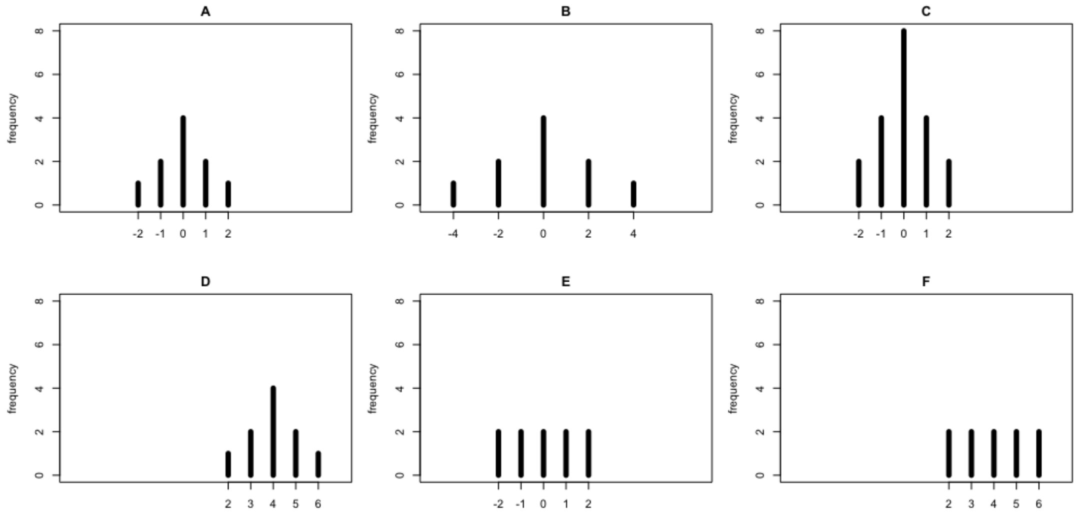
You should have a solid foundation in descriptive statistics as provided in a typical statistics course in an undergraduate economics or management program. Besides the basic concepts and formulas (central tendency, (co-) variance, simple plots, representation of distributions) you should be able to apply theoretical knowledge to observed distributions (as shown in the example tasks).

Literature:

- Statistics for Business and Economics; D. Anderson, D. Sweeney, T. Williams, J. Freeman, E. Shoesmith; Cengage 2017; Chapter 2 and 3
- Statistische Methoden der VWL und BWL – Theorie und Praxis; J. Schira; Pearson 2021; Chapter 1 - 3

Example Tasks:

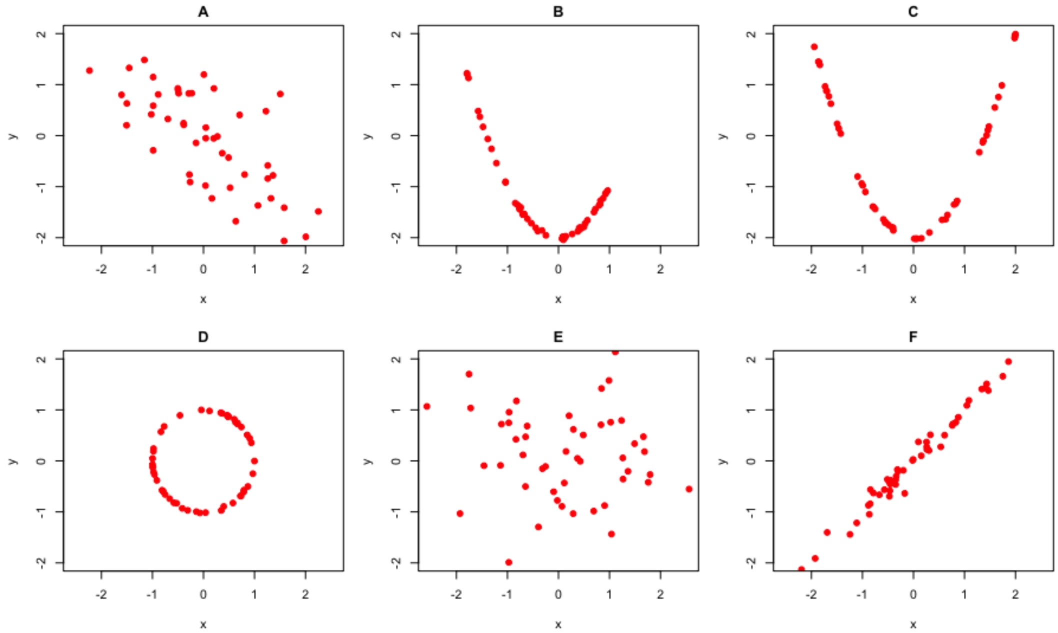
(E) The diagram illustrates six different discrete distributions (A–F). Distribution A has a variance of 1.2.



For the other distributions (B-F) choose a variance from the following list of values using plausibility considerations (multiple assignments are possible).

0, 1, 1.2, 2, 4.8, 6, “none of them”

(F) The diagram illustrates six different scatter plots (A–F).



Assign to each plot a correlation coefficient from the following list (multiple assignments are possible).

-1.2, -0.95, -0.7, 0, 0.7, 0.95, 1

IV. Stochastic

probability theory (random variables, distribution), convergence (central limit theorem, law of large numbers)

You should have a solid understanding of random variables and their characteristics (expected value, variance, and distribution). You should know the convergence concepts.

Example Tasks:

(G) Consider a game of dice with the following payoffs.

Dice score	Payoff
⚀	2
⚁	0
⚂	-1
⚃	0
⚄	0
⚅	2

a. What is the expected payoff when you play one round?
b. What is the variance of the payoffs in one round?
c. Now consider you increase all payoffs by 1. What is the new variance for the payoffs?
   (a) Larger than before
   (b) Smaller than before
   (c) The same as before
   (d) Unknown

(H) Consider the sample average $\frac{1}{n}\sum_{i=1}^{n} x_i$, which is based on a random sample of n observations (sampling with replacement) from a population of 1 000 individuals, as an unbiased estimator for the population mean. Mark the following statements as true or false.

	true/false/uncertain
"Increasing the sample size directly leads to a smaller difference between the sample mean and the population mean."	
"The distribution of the estimator is wider (higher variance) with a small sample size than with a larger sample size."	

“With very large sample sizes, it is unlikely to get an estimate that is far from the population mean.”	
“With sample size n=1000, the sample mean is always equal to the population mean.”	
“For an infinite sample size, the sample mean equals the population mean.”	

V. Inductive Statistics

estimation, confidence interval, hypothesis testing

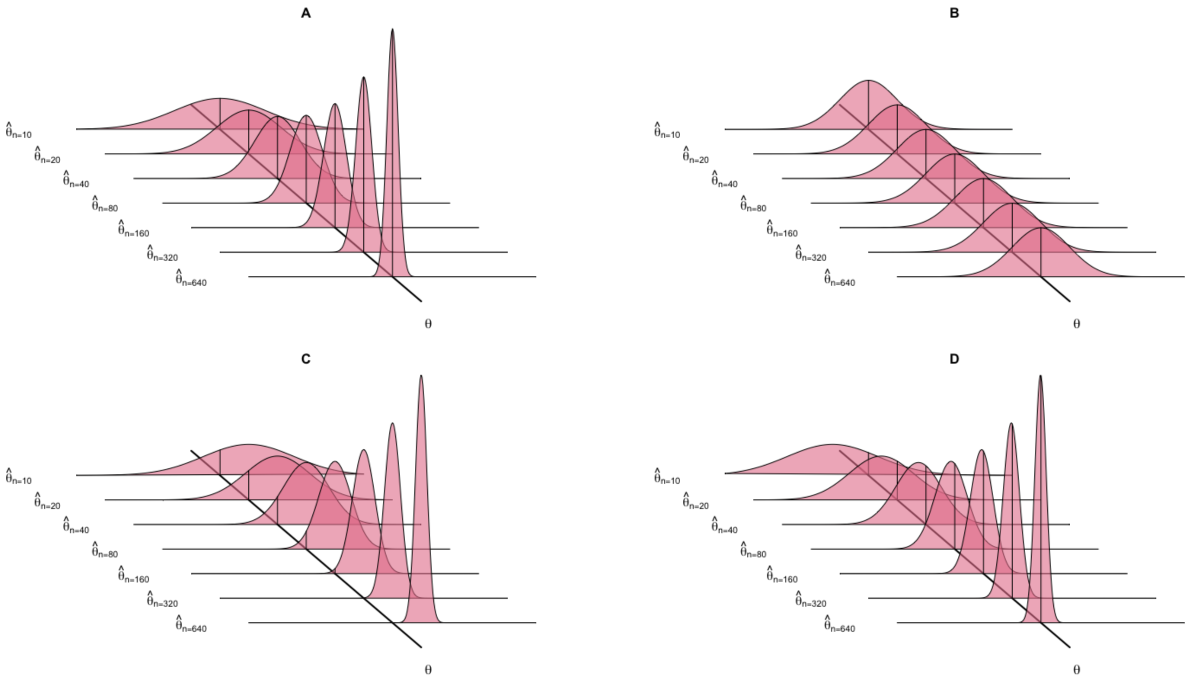
You should be familiar with desirable properties of an estimator. You should be able to compute point- and interval estimates for simple features (e.g., population mean; material like formulas and critical values are provided if necessary). You should know the concept of standard errors and their interpretation.

Literature:

- Statistics for Business and Economics; D. Anderson, D. Sweeney, T. Williams, J. Freeman, E. Shoesmith; Cengage 2017; Chapter 7 - 9

Example Tasks:

(I) The following four graphs illustrate different estimators for a parameter θ. Each density shows the distribution of the estimator based on a given sample size, e.g., $\hat{\theta}_{n=100}$ is the distribution for sample size n=100. Small sample sizes are in the background, large samples in the foreground. The solid line marked with θ shows the true parameter.



Mark with “X” if an estimator (A-D) is unbiased or consistent respectively:

	unbiased	consistent
A		
B		
C		
D		

(J) A hypothesis test (with $\alpha = 0.05$) leads to a p-value of 0.10. Mark the following statements as true or false.

	true/false
“The null hypothesis is proven to be true.”	
“The null hypothesis will be rejected.”	
“The null hypothesis won’t be rejected.”	
“The alternative hypothesis will be rejected.”	
“Under the null hypothesis, the realized test statistic has a probability of 0.10.”	
“It is possible that the true parameter differs from the null hypothesis.”	
“Even if the p-value were 0.01, it’s possible that the null hypothesis is actually true.”	

VI. Multiple Linear Regression Analysis

requirements, estimation, testing, Interpretation

Regression analysis is a central method in econometrics. You should have a solid knowledge regarding the linear regression model and the requirements (homoscedasticity, exogeneity).

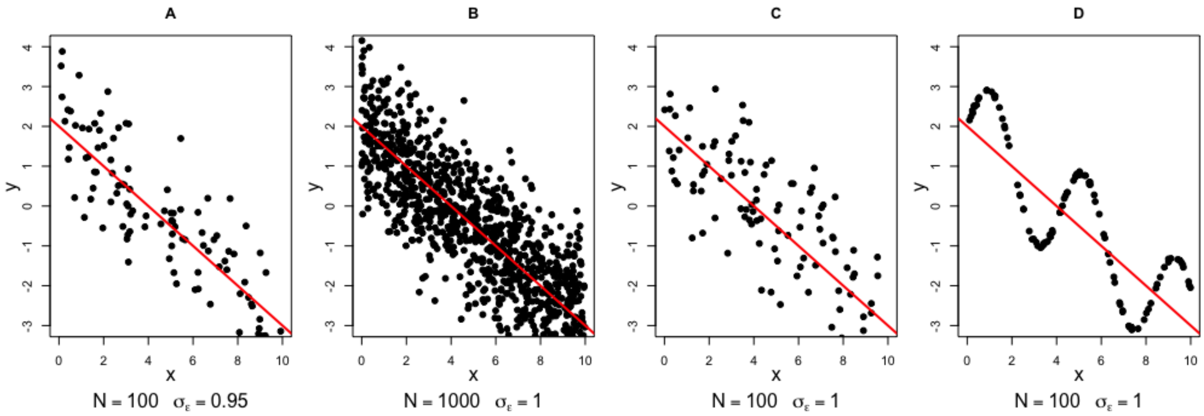
Literature:

- Introductory Econometrics – a modern Approach; J. Wooldridge; Cengage 2018; Chapter 2 - 4

(K) For the samples below, a regression analysis is carried out to estimate the model:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

The regression line is drawn in red. The respective sample sizes and standard deviations of the residuals are underneath each figure.



Which sample leads to the smallest standard error for the estimate of β_1?

(L) Assume the price of a used car is determined by two variables, age (in years) and mileage (in km). On average, the price decreases by 1 000€ for each year (holding everything else constant). In addition, mileage has also a negative effect on the price. The variable age and mileage have a positive relationship, i.e., the older a car the higher the mileage (on average).
You estimate the following regression model based on a representative sample.

$$price_i = \beta_0 + \beta_1 age_i$$

What is the value you expect for β_1? (Choose a plausible value from the list.)

-1200, - 1000, -800, 0, 800, 1000, 1200

VII. Microeconomics

economic foundations (supply & demand, cost & profit, utility, preferences, production function), strategic interaction (non-cooperative games)

Microeconomics studies the behaviour of individuals and firms in decision making and resource allocation. Students should have a solid knowledge of the standard microeconomic concepts (preferences, cost & profit, supply & demand, strategic interaction) typically taught in an intermediate microeconomics course of an undergraduate program in economics and management.

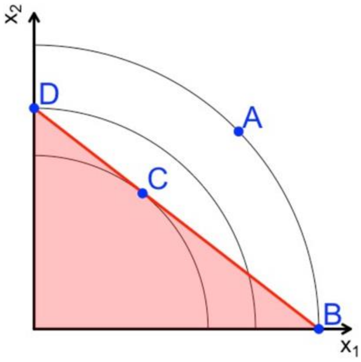
Literature:

- Intermediate Microeconomics: A Modern Approach; H. Varian; Norton & Company 2019; Chap 1-12

- Games of Strategy; A. Dixit, S. Skeath, D. McAdams; Norton & Company 2020; Chap. 3-5 and 7

Example Tasks:

(M) The diagram shows the indifference curves of an ordinary consumer in a two-product setup (x_1, x_2) and the budget constraint (red line).



Which product-bundle (A-D) will the consumer choose?

(N) Simon and Laura want to go out for dinner. Simon prefers pasta, Laura prefers potatoes. Both would love to go out to eat together. There is no restaurant in your city that offers both pasta and potatoes. There is only a pasta house and a potato house. The table shows the payoffs (Simon / Laura).

		Laura	
		potatoes	pasta
Simon	potatoes	2 / 4	0 / 0
	pasta	1 / 1	4 / 2

If possible, determine the Nash equilibrium in pure strategies (a-e). If there is more than one equilibrium, just give one of them.

a. Simon = potatoes / Laura = potatoes
b. Simon = potatoes / Laura = pasta
c. Simon = pasta / Laura = potatoes
d. Simon = pasta / Laura = pasta
e. none of these

VIII. Algorithms

structure (controls, data types), reading skills

You should know what an algorithm is and what its structural elements are (loops, branches, variables). You should be able to run simple algorithms "by hand" and understand their working principle. It is not necessary to have coding skills in a concrete programming language.

Literature:

- Introduction to algorithms; T. Cormen, C. Leiserson, R. Rivest, C. Stein; The MIT Press 2022; Chapter 1-3

Example Tasks:

(O) Run the following algorithm by hand.

```
S = 9
b = S
n = 0
while n ≤ 2 do
    b = ½(b + S/b)
    n = n + 1
end
```

Give the final value of the following variables S, b and n.

(P) Consider the following algorithm.

```
A (a list of numbers, at least one element)
B = length of list A
C = 1
for i = 1 to B do
    if A[i] > A[C] then
        C = i
    end
end
return C
```

a. What is the algorithm doing?
   i. sorting the list (ascending order)
   ii. searching the smallest element in the list
   iii. searching the position of the largest element in the list
   iv. computing the median for the list
   v. none of these
b. Consider that the list is of length N and the runtime takes D units of time. How will the runtime change (D_{new}) when the list is now of length $N_{new} = 2N$
   i. $D_{new} < D$
   ii. $D_{new} \approx D$
   iii. $D_{new} \approx \sqrt{D}$
   iv. $D_{new} \approx 2D$
   v. $D_{new} \approx D^2$